*Laurent series*

Consider a function f (z) that is not analytic at the points z0 , z1, z2 , z3 , …, zk .Then we expand the function f (z) at any point say z0  in a series which contains both positive and negative powers of z- z∞ . This series is called as the Laurent series of the f (z).

Consider power series of the form

 

 

Let radius of convergence of the series (1) is ‘r1’ and this series converges to some analytic function f1(z)

 f1 (z) = , |z z0| < r1

Let  =  then (2)   which is a power series in  and converges to some analytic function  within its circle of convergence. Let its radius of convergence  .

 . f2 (z) =  , |z – z0| > r2

Thus the region of convergence of the series (2) is the region exterior to the circle |z – z0| > r2. Let r1 < r2. Then the intersection of two regions of convergence is r1 >|z – z0| > r2  . Therefore the series of positive & negative powers of (z –z0) converges in r1 >|z – z0| > r2 to analytic function f1(z) + f2(z) i.e. f (z) = f1(z) + f2(z) = , r1 >|z – z0| > r2  . This series is called as Laurent series.

**Laurent Series :-**

Let C1 and C2 denote the concentric circles |z – z0| = r1 and |z – z0| = r2  respectively with r1 < r2 . Let f (z) be analytic in a region containing the circular annulus r1 < |z – z0| < r2  . Then f (z) be represented as convergent series of positive and negative powers of z -z0  given by

f(z) =  + 

Outer radius =C2

Inner radius = C1

Where an =  , bn  = 

Note:

1. The Coefficients of the positive powers of. In the Laurent series cannot be replaced by the derivative expressions  , although they are identical in form as in Taylor Series, since f (z) is not analytic throughout the region inside c2 & the Cauchy integral formula for derivatives cannot be used.
2. In the Laurent series let r1 .0. Then f (z) is analytic in |z – z0| < r2 except at z0.

**∴** The region of convergence is 0 < |z – z0 | < r2 . If f (z) is analytic at z0 also then the Laurent

Series is the same as Taylor Series.

1. In Laurent Series, let r2 ∞, r then the region of convergence is |z – z0| < r1.
2. Laurent Series expansion of f (z) in the annulus region r1 > |z – z0| > r2  is unique.

Example 1: Obtain the Laurent series expansion of the following series about the given point f(z) =  Solution: f (z) =   Singular points are 1 & 2

2

1

∴ Possible regions are | z | < 1, 1 < | z | < 2 & | z | > 2. f (z) is analytic within the region | z | < 1, 1 < | z | < 2 & | z | > 2

f (z) =  =  =  = 

=  = 

∴ f (z) has Taylor expansion in region | z | < 1

+

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